Math 115

Summer 2017

Lecture 17

Find all excluded values:

1) $\frac{2x+5}{25x^2-4}$

Deno. = 0

Solve

$25x^2-4=0$

$(5x+2)(5x-2)=0$

$x=-\frac{2}{5}$ $x=\frac{2}{5}$

E.V. $\pm\frac{2}{5}$

2) $\frac{3x-7}{3x^2-4x-7}$

$3x^2-4x-7=0$

$a=3, b=-4, c=-7$

$b^2-4ac=(-4)^2-4(3)(-7)$

$=16+84=100$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{4\pm10}{6}$

$x=\frac{14}{6}=\frac{7}{3}$ $x=\frac{-6}{6}=-1$

E.V. $\frac{7}{3}, -1$

Simplify:

1) $\dfrac{2xy+2x-3y-3}{2xy+4x-3y-6}$

$= \dfrac{2x(y+1)-3(y+1)}{2x(y+2)-3(y+2)}$

$= \dfrac{(y+1)(2x-3)}{(y+2)(2x-3)} = \boxed{\dfrac{y+1}{y+2}}$

2) $\dfrac{4x^2+6x+9}{8x^3-27}$

$= \dfrac{4x^2+6x+9}{(2x)^3-3^3}$

$= \dfrac{1\ 4x^2+6x+9}{(2x-3)(4x^2+6x+9)}$

$= \boxed{\dfrac{1}{2x-3}}$

Simplify:

1) $\dfrac{x^2-4x+4}{x^2-4} \cdot \dfrac{x^2+3x}{x^2-2x}$

$= \dfrac{(x-2)(x-2)}{(x+2)(x-2)} \cdot \dfrac{x(x+3)}{x(x-2)}$

$= \boxed{\dfrac{x+3}{x+2}}$

2) $\dfrac{x^2+7x+10}{x-1} \div \dfrac{x^2+2x-15}{x^2-4x+3}$

$= \dfrac{(x+2)(x+5)}{x-1} \cdot \dfrac{(x-1)(x-3)}{(x+5)(x-3)}$

$= \dfrac{x+2}{1} = \boxed{x+2}$

Find LCM:

1) x^2-x-30 , x^2-36

$(x-6)(x+5)$ $(x-6)(x+6)$

$\boxed{(x-6)(x+5)(x+6)}$

2) x^2-25, $x^2+10x+25$, $3x^3-15x^2$

$x^2-25 = (x+5)(x-5)$

$x^2+10x+25 = (x+5)^2$

$3x^3-15x^2 = 3x^2(x-5)$

$LCM = 3x^2(x+5)^2(x-5)$

Simplify

1) $\dfrac{3x-1}{x^2+5x-6} - \dfrac{2x-7}{x^2+5x-6} = \dfrac{3x-1-2x+7}{x^2+5x-6} = \dfrac{1}{(x+6)(x-1)}$

$= \boxed{\dfrac{1}{x-1}}$

2) $\dfrac{2x^2}{x-5} - \dfrac{25+x^2}{x-5}$

$= \dfrac{2x^2-25-x^2}{x-5} = \dfrac{x^2-25}{x-5} = \dfrac{(x+5)(x-5)}{x-5} = \boxed{x+5}$

Simplify:

$$\frac{2x}{x-7} - \frac{x}{x+3} = \frac{2x(x+3)}{(x-7)(x+3)} - \frac{x(x-7)}{(x+3)(x-7)}$$

$$= \frac{2x^2+6x-x^2+7x}{(x-7)(x+3)}$$

$$= \frac{x^2+13x}{(x-7)(x+3)} \quad \frac{x(x+13)}{(x-7)(x+3)}$$

$$\frac{3}{x+5} - \frac{1}{x-5}$$

$$= \frac{3(x-5)-1(x+5)}{(x+5)(x-5)}$$

$$= \frac{2x-20}{(x+5)(x-5)}$$

Simplify:

$$\frac{12}{x^2-5x+6} - \frac{5}{x^2-9}$$

$$= \frac{12(x+3)}{(x-3)(x-2)(x+3)} - \frac{5(x-2)}{(x+3)(x-3)(x-2)}$$

$$= \frac{12x+36-5x+10}{LCD} = \frac{7x+46}{(x+3)(x-3)(x-2)}$$

$$\frac{x+10}{x^2-3x-4} - \frac{8}{x^2+6x+5} - \frac{9}{x^2+x-20}$$

$$= \frac{(x+10)(x+5)}{(x-4)(x+1)(x+5)} - \frac{8(x-4)}{(x+5)(x+1)(x-4)} - \frac{9(x+1)}{(x+5)(x-4)(x+1)}$$

$$= \frac{x^2+5x+10x+50-8x+32-9x-9}{(x-4)(x+1)(x+5)} = \boxed{\frac{x^2-2x+73}{(x-4)(x+1)(x+5)}}$$

$$\frac{x^2-4}{x^2-9} \cdot \frac{x+3}{x-2} - \frac{3}{x^2-4x+3}$$

$$= \frac{(x+2)\cancel{(x-2)}}{\cancel{(x+3)}(x-3)} \cdot \frac{\cancel{x+3}}{\cancel{x-2}} - \frac{3}{x^2-4x+3}$$

$$= \frac{x+2}{x-3} - \frac{3}{x^2-4x+3} = \frac{(x+2)(x-1)}{(x-3)(x-1)} - \frac{3}{(x-3)(x-1)}$$

$$= \frac{x^2-x+2x-2-3}{(x-3)(x-1)}$$

$$= \boxed{\frac{x^2+x-5}{(x-3)(x-1)}}$$

Find LCD, E.V., then Solve

$\frac{x}{3} + \frac{4}{5} = \frac{2}{15}$ LCD $= 3 \cdot 5 = 15$

E.V.: None

$\cancel{15}^{5} \cdot \frac{x}{\cancel{3}} + \cancel{15}^{3} \cdot \frac{4}{\cancel{5}} = \cancel{15} \cdot \frac{2}{\cancel{15}}$

$5x + 12 = 2 \Rightarrow 5x = 2 - 12$ $5x = -10$

$\{-2\}$ $\boxed{x = -2}$

$\frac{x+4}{4} - \frac{x-3}{3} = \frac{11}{12}$ LCD $= 12$

E.V.: None

$\cancel{12}^{3} \cdot \frac{x+4}{\cancel{4}} - \cancel{12}^{4} \cdot \frac{x-3}{\cancel{3}} = \cancel{12} \cdot \frac{11}{\cancel{12}}$

$3(x+4) - 4(x-3) = 11$

$3x + 12 - 4x + 12 = 11$

$-x + 24 = 11$ $-x = 11 - 24$ $-x = -13$

$\{13\}$ $\boxed{x = 13}$

$8 + \frac{7}{x} = x + 2$ LCD = x

E.V.: 0

$8x + x \cdot \frac{7}{x} = x \cdot x + 2x$

$8x + 7 = x^2 + 2x$

$x^2 - 6x - 7 = 0$

$x^2 - 6x + 9 = 7 + 9$

$(x-3)^2 = 16$

$x - 3 = \pm\sqrt{16}$

$x - 3 = \pm 4$

$x = 3 \pm 4$

$x = 7$, $x = -1$

$\{-1, 7\}$

Solve $\frac{4x}{x^2+x-30} + \frac{2}{x-5} = \frac{1}{x+6}$

LCD = $(x+6)(x-5)$ E.V.: $-6, 5$

$4x + 2(x+6) = 1(x-5)$

$4x + 2x + 12 = x - 5$

$6x - x = -5 - 12$ $5x = -17$

$x = \frac{-17}{5}$

Solve: $x + \frac{14}{x-2} = \frac{7x}{x-2} + 1$

LCD $= x-2$ E.V.: 2

$x(x-2) + 14 = 7x + 1(x-2)$

$x^2 - 2x + 14 = 7x + x - 2$

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0 \Rightarrow \boxed{x=8}$, ~~$x=2$~~

$x^2 - 10x + 25 = -16 + 25$ → $x-5 = \pm\sqrt{9}$

$(x-5)^2 = 9$ → $x-5 = \pm 3$ → $x = 5 \pm 3$

$\boxed{x=8}$, ~~2~~

Solve $x + \frac{x}{x-5} = \frac{5}{x-5} - 7$

LCD $= x-5$ E.V.: 5

$x(x-5) + x = 5 - 7(x-5)$

$x^2 - 5x + x = 5 - 7x + 35$

$x^2 + 3x - 40 = 0$

$a=1 \quad b=3 \quad c=-40$

$b^2 - 4ac = 9 + 160 = 169$

$x = \frac{-3 \pm \sqrt{169}}{2} = \frac{-3 \pm 13}{2}$

$x = \frac{-3+13}{2} = ~~5~~$

$x = \frac{-3-13}{2} = -8$

$\{-8\}$

Solve

$$\frac{x}{2x+2} + \frac{2x-16}{4x+4} = \frac{2x-3}{x+1}$$

$2(x+1)$ $\quad 4(x+1)$

$LCD = 4(x+1)$ $\quad$ E.V. -1

$$\cancel{4}^{2}\cancel{(x+1)} \cdot \frac{x}{\cancel{2}(x+1)} + \cancel{4(x+1)} \cdot \frac{2x-16}{\cancel{4(x+1)}} = \cancel{4(x+1)} \cdot \frac{2x-3}{\cancel{x+1}}$$

$$2x + 2x - 16 = 4(2x-3)$$

$$4x - 16 = 8x - 12$$

$$4x - 8x = -12 + 16$$

$$-4x = 4$$

$x = -1$ ✗

$\emptyset$

Solve

$$\frac{2x+3}{x-1} - \frac{2}{x+3} = \frac{5-6x}{x^2+2x-3}$$

$LCD = (x-1)(x+3)$ $\quad$ E.V.: $1, -3$

$$(2x+3)(x+3) - 2(x-1) = 5-6x$$

$$2x^2 + 9x + 9 - 2x + 2 - 5 + 6x = 0$$

$$2x^2 + 13x + 6 = 0 \Rightarrow (2x+1)(x+6) = 0$$

$a = 2$ $\quad b = 13$ $\quad c = 6$

$$b^2 - 4ac = 13^2 - 4(2)(6) = 169 - 48 = 121$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-13 \pm \sqrt{121}}{2(2)}$$

$$= \frac{-13 \pm 11}{4}$$

$x = \frac{-13+11}{4} = \frac{-2}{4} = -\frac{1}{2}$ ✓

Not among E.V.

$x = \frac{-13-11}{4} = \frac{-24}{4} = -6$ ✓

$\left\{-\frac{1}{2}, -6\right\}$

If b^2-4ac is a perfect square, then ax^2+bx+c can be factored.

Complex Fractions:

It is a fraction that contains at least one fraction.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}}, \quad \frac{4+\frac{2}{3}}{1-\frac{1}{4}}, \quad \frac{x-\frac{3}{x}}{1+\frac{2}{x}}, \ldots$$

To Simplify,

Multiply everything by LCD, Simplify

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}} = \frac{\overset{3}{\cancel{6}}\cdot\frac{1}{\cancel{2}}-\overset{2}{\cancel{6}}\cdot\frac{1}{\cancel{3}}}{\cancel{6}\cdot\frac{5}{\cancel{6}}} = \frac{3-2}{5} = \boxed{\frac{1}{5}}$$

LCD = 6

$$\frac{1+\frac{2}{x}}{x-\frac{4}{x}} = \frac{x\cdot 1+\cancel{x}\cdot\frac{2}{\cancel{x}}}{x\cdot x-\cancel{x}\cdot\frac{4}{\cancel{x}}} = \frac{x+2}{x^2-4}$$

LCD = x

$$= \frac{1(x+2)}{(x+2)(x-2)}$$

$$= \boxed{\frac{1}{x-2}}$$

Simplify

$$\frac{x+7+\frac{12}{x}}{1+\frac{3}{x}} = \frac{x^2+7x+12}{x+3}$$

LCD = x

$$= \frac{(\cancel{x+3})(x+4)}{\cancel{x+3}} = \boxed{x+4}$$

$$\frac{\frac{x}{4}-\frac{4}{x}}{1-\frac{4}{x}} = \frac{\cancel{4x}\cdot\frac{x}{\cancel{4}}-\cancel{4x}\cdot\frac{4}{\cancel{x}}}{4x\cdot 1-\cancel{4x}\cdot\frac{4}{\cancel{x}}} = \frac{x^2-16}{4x-16} = \frac{(x+4)(\cancel{x-4})}{4(\cancel{x-4})}$$

LCD = 4x

$$= \boxed{\frac{x+4}{4}}$$

Simplify

$$\frac{\frac{2}{x+5}+\frac{4}{x+3}}{\frac{6x+26}{x^2+8x+15}}$$

$LCD = (x+5)(x+3)$

$$= \frac{2(x+3)+4(x+5)}{6x+26}$$

$$= \frac{2x+6+4x+20}{6x+26}$$

$$= \frac{6x+26}{6x+26} = \boxed{1}$$

Simplify

$$\frac{\frac{2}{x+2}-\frac{6}{x+7}}{\frac{-4x+2}{x^2+9x+14}}$$

$LCD = (x+2)(x+7)$

$$= \frac{2(x+7)-6(x+2)}{-4x+2}$$

$$= \frac{-4x+2}{-4x+2}$$

$$= \boxed{1}$$

$$\frac{x+1}{x-x^{-1}} = \frac{x+1}{x-\frac{1}{x}} = \frac{x^2+x}{x^2-1}$$

$$\frac{2-\frac{1}{x}-\frac{1}{x^2}}{1-\frac{3}{x}+\frac{2}{x^2}}$$ LCD = x, LCD = x^2

$$= \frac{x(x+1)}{(x+1)(x-1)} = \frac{x}{x-1}$$

$$= \frac{2x^2-x-1}{x^2-3x+2} = \frac{(2x+1)(x-1)}{(x-2)(x-1)} = \boxed{\frac{2x+1}{x-2}}$$

Simplify:

$$\frac{5x^{-1}-2y^{-1}}{25x^{-2}-4y^{-2}} = \frac{\frac{5}{x}-\frac{2}{y}}{\frac{25}{x^2}-\frac{4}{y^2}} = \frac{x^2y^2\cdot\frac{5}{x}-x^2y^2\cdot\frac{2}{y}}{x^2y^2\cdot\frac{25}{x^2}-x^2y^2\cdot\frac{4}{y^2}}$$

Hint: Convert − exponent to + exponent

LCD = x^2y^2 $= \frac{5xy^2-2x^2y}{25y^2-4x^2}$

$(5y)^2-(2x)^2$

$$= \frac{xy(5y-2x)}{(5y+2x)(5y-2x)} = \boxed{\frac{xy}{5y+2x}}$$

John can install the carpet in a room in 3 hrs.

Mike can do the same job in 5 hrs.

How long if they work together?

Work by John + Work by Mike = 1 Complete work

Rate · time + Rate · time = 1

$\frac{1}{3} \cdot t + \frac{1}{5} \cdot t = 1$

$\frac{t}{3} + \frac{t}{5} = 1$

LCD = 15

$8t = 15$

$t = \frac{15}{8}$

$\frac{15}{8}$ hrs

Michelle can do a job alone in 8 days.

Michelle and Laura can do the same job together in 5 days.

How long does it take Laura to do the job alone?

Work by Michelle + Work by Laura = 1 Complete work

$\frac{5}{8} + \frac{5}{x} = 1$

LCD = $8x$

$\frac{40}{3}$ days

$\frac{1}{8} \cdot 5 + \frac{1}{x} \cdot 5 = 1$

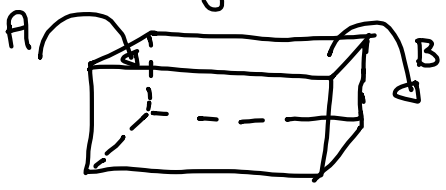
Rate · time + Rate · time

$5x + 40 = 8x$

$3x = 40$

$x = \frac{40}{3}$

Pipe A can fill up an empty pool in 6 hrs. Pipe B can empty a full pool in 10 hrs. If the pool is empty and both pipes working, how long does it take to fill up the pool?



work by A − work by B = 1 comp. work

$\frac{1}{6}\cdot t - \frac{1}{10}\cdot t = 1$

$5t - 3t = 30$

$2t = 30$

$t = 15$ → 15 hrs

$\frac{t}{6} - \frac{t}{10} = 1$

LCD = 30

It takes Lisa 3 times longer than Mary to do a certain job. Together, they can do it in 21 days. How many days to do the work alone for each one?

Mary → x days → Rate $\frac{1}{x}$

Lisa → $3x$ days → Rate $\frac{1}{3x}$

work by Mary + work by Lisa = 1 comp. work

$\frac{1}{x}\cdot 21 + \frac{1}{3x}\cdot 21 = 1$

$\frac{21}{x} + \frac{21}{3x} = 1$

$\frac{21}{x} + \frac{7}{x} = 1$

LCD = x

$21 + 7 = x$

$x = 28$

Lisa: 84 days

Mary: 28 Days

John takes 9 hours longer than Jim to do a job.

Together, they do the job in 6 hrs.

How long if working alone?

Jim → x hours

John → $x+9$ hours

$x^2 - 3x - 54 = 0$

$(x+6)(x-9) = 0$

~~$x=-6$~~ $x=9$

Jim 9 hrs, John 18 hrs

$\frac{1}{x} \cdot 6 + \frac{1}{x+9} \cdot 6 = 1$

$\frac{6}{x} + \frac{6}{x+9} = 1$

LCD $= x(x+9)$

$6(x+9) + 6x = x(x+9)$

$6x + 54 + 6x = x^2 + 9x$

Agenda for tomorrow:

1) Collect SG 19 @ 6:00 AM

2) Lecture & Work on SG 20

3) Final exam starts @ 7:30 AM.

$d = r \cdot t$

speed, time → Distance

50 mph for 2.5 hrs

$d = r\,t$

$= 50(2.5) = 125$ Miles

Patrick drove 12 miles in the same time that SpongeBob drove 18 miles.

	d	r	t
SpongeBob	18	r_1	t
Patrick	12	r_2	t

$18 = r \cdot t$

$t = \frac{18}{r_1}$

$t = \frac{12}{r_2}$

SpongeBob drove 3 mph faster than Patrick.

	d	r	t
SpongeBob	18	$x+3$	t
Patrick	12	x	t

$t = \frac{18}{x+3}$

$t = \frac{12}{x}$

Same time $\frac{18}{x+3} = \frac{12}{x}$

Patrick 6 MPH

SpongeBob 9 MPM

Cross-Multiply

$18x = 12(x+3)$

$18x = 12x + 36$

$6x = 36$

$x = 6$

John drove 160 miles in the same time that Jack drove 100 miles.

Jack was driving 30 mph slower than John.

	d	r	t
John	160	x	t
Jack	100	$x-30$	t

$t = \frac{160}{x}$

$t = \frac{100}{x-30}$

$\frac{\cancel{160}^{8}}{x} = \frac{\cancel{100}^{5}}{x-30}$

$8(x-30) = 5x$

$8x - 240 = 5x$

$3x = 240$

$x = 80$

John → 80 MPH

Jack → 50 MPH